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From the last term of (1) in a similar manner

$$\lim_{n=\infty} n \sin^{-1} \left[ \frac{a \tan \frac{\pi}{n}}{(R^2 - a^2)^{1/2}} \right] = \frac{\pi a}{(R^2 - a^2)^{1/2}}$$

Substituting these values in (1) and reducing

$$V_{II} = \lim_{n=\infty} V = \frac{4}{3} \pi [R^3 - (R^2 - a^2)^{3/2}]$$

which is the result for the familiar problem of the volume cut from a sphere of radius  $R$  by circular cylinder of radius  $a$  when the center of the sphere is on the axis of the cylinder.

Also solved by PAUL CAPRON, A. W. SMITH and the PROPOSER.

**401. Proposed by LAENAS G. WELD, Pullman, Ill.**

Given a continuum of triangles whose sides are in arithmetical progression, the common difference being  $h$ : (a) The ratio of the mean value of all the triangles, the mean of whose three sides is not greater than  $\mu$ , to the area of the triangle, the mean of whose three sides is equal to  $\mu$ , is  $(\mu + 2h)/3\mu$ . Indicate the limiting values of this ratio and show that, when it is equal to  $1/2$ , the triangle whose mean side is  $\mu$  is right angled. (b) The ratio of the mean value of the areas of the circles inscribed in all these triangles, the mean of whose three sides is not greater than  $\mu$ , to that of the circle inscribed in the triangle, the mean of whose three sides is equal to  $\mu$ , has the limiting values  $1/2$  and  $1/3$ . When the triangle whose mean side is  $\mu$  is right angled, the ratio in question is  $4/9$ . (c) Of the circles circumscribed about these triangles the minimum has the radius  $2h$ .

**SOLUTION BY A. H. WILSON, Haverford College.**

Let  $x - h$ ,  $x$ , and  $x + h$  be the three sides. Then the area is

$$[s(s-a)(s-b)(s-c)]^{1/2} = \frac{1}{4}[3x^2(x^2 - 4h^2)]^{1/2}.$$

The mean area of all triangles, the mean of whose three sides is not greater than  $\mu$ , is

$$\frac{\sqrt{3}}{4(\mu - 2h)} \int_{2h}^{\mu} x \sqrt{x^2 - 4h^2} dx = \frac{\sqrt{3}}{12} \sqrt{\mu^2 - 4h^2} (\mu + 2h),$$

the least value of  $x$  being  $2h$ .

The area of the triangle for which  $x = \mu$  is

$$\frac{1}{4}[3\mu^2(\mu^2 - 4h^2)]^{1/2} = \frac{\sqrt{3}}{4} \mu(\mu^2 - 4h^2)^{1/2}.$$

(a) The ratio of these two areas is  $(\mu + 2h)/3\mu$ . The limits of this ratio occur for the values  $\mu = 2h$  and  $\mu = \infty$ , and are, respectively,  $2/3$  and  $1/3$ . If the ratio is equal to  $1/2$ , then  $\mu = 4h$ ; and the sides  $3h$ ,  $4h$ , and  $5h$  are those of a right triangle.

(b) The area of the inscribed circle is

$$\pi r^2 = \pi(x^2 - 4h^2)/12. \quad (r = [(s-a)(s-b)(s-c)/s]^{1/2}).$$

The mean of such areas for triangles of the first class is

$$\frac{\pi}{12(\mu - 2h)} \int_{2h}^{\mu} (x^2 - 4h^2) dx = \frac{\pi}{36} (\mu^2 + 2h\mu - 8h^2).$$

The area of the circle of the second class is  $\pi(\mu^2 - 4h^2)/12$ ; and the ratio of the two areas is  $(\mu + 4h)/3(\mu + 2h)$ . The limiting values of this ratio are  $1/2$  and  $1/3$ , and for the value  $4/9$  the mean side is  $4h$ , and the triangle is right-angled.

(c) The area of the circumscribed triangle is  $\pi R^2 = \pi(abc)^2/16s(s-a)(s-b)(s-c) = \pi(x^2 - h^2)^2/3(x^2 - 4h^2)$ . For a minimum, equate the derivative to 0; and there results for  $x$  the value  $\sqrt{7}h$ , giving a minimum. For this value of  $x$  the radius of the circumscribed circle is  $2h$ .

Also solved by ELIJAH SWIFT.